*1. Report a good choice for single-parameter likelihood for the number of cars you sell in one day (xi), and an informative conjugate prior for the parameter.*

A good choice for single-parameter likelihood for the number of cars I sell in one day (xi) is **Poisson likelihood**, which indicates a discrete probability distribution that expresses the probability of a given number of events occurring in a fixed interval of time.

An informative conjugate prior for the parameter would be a Gamma Prior,

, whose variance, mean

To make the variance =1, and mean=1 because of our primary estimate of 1 car/day, we can get α=1, β=1.

Consequently, the prior is

, which is an informative prior as ∫g(θ)dθ<∞ as required.

*2. After one week, you sell 4 cars. Report the posterior distribution of your parameter.*

When we evaluate from a time interval of one week, which is 5 days, according to the summation property of gamma distribution[[1]](#footnote-1), we have the 5-day prior

Because, according to the property “Sum of independent Poisson random variables is Poisson”, [[2]](#footnote-2)we know for one week, total car sales number y

Thus, for the conjugate gamma prior and Poisson likelihood, we have posterior in weekly view:

*3. Use the posterior predictive distribution to calculate (or simulate) the probability that you will match or exceed Ronald Aylmer’s 2-week performance. Report this probability (rounded to nearest percent). Assume that your ability to sell cars remains constant throughout your employment.*

In order to match or exceed Ronald Aylmer’s 2-week performance, which is 10 cars in 2 week, I have to sell no less than 10-4=6 cars in week 2, which means y2>=6|y1=4.

The Posterior Predictive Distribution is:

Where,and , α=9,β=2. And

Let , according to the Law of Large Numbers,

The empirical distribution of

Consequently, using Monte Carlo Simulation, we first generate y(i) using rgamma function in R, then compute the g(y(i)), and finally use the expectation of g(y(i) to calculate the probability of y2>=6.

When S=100, p=12%;

When S=1000, p=18.3%

When S=10000, p=15.71%

When S=100000, p=15.393%

When S=500000, p=15.4178%

When S=1000000, p= 15.5311%

Consequently, we get the final result of **a probability 15%** to match or exceed Ronald’s 2-week performance.

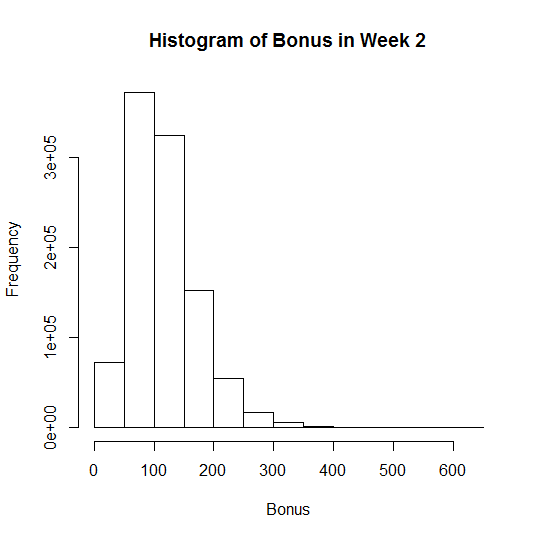
In fact, I also calculated the theoretical probability of the question, which can be acquired through the integral, and the result is 15.5027%.

*4. What is the probability that your bonus after your second week of work, b2, will exceed $100, given that you sold 4 cars in week 1? Report this to the nearest percent. Plot the distribution of b2 as a histogram, not a density.*

Given the fact that I sold 4 cars in week 1, I have a posterior of , which is the same as in problem 3.

In order that

There has to be a y with . Because y has to be integer, y2>=5. The result is shown in the following histogram and the corresponding probability of getting >100 bonus is 33%.



##################Code######################

######problem 3###########

theta=seq(0,10,length=11);alpha=9;beta=2;S=1000000;

posterior <- dgamma(theta,alpha,beta)

expectation <- function(x){

z=dgamma(x,9,2)\*as.numeric(x>=6)

return (z)

}

theoretical=integrate(expectation,lower=0,upper=Inf)

expectation <- sum(as.numeric(rgamma(S,alpha,beta)>=6))/S

print(expectation)

#######Problem 4############

threshold <- ceiling(100^(1/sqrt(2))/(2\*pi))

temp <- rgamma(S,alpha,beta)

expectation2 <- sum(as.numeric(temp>=threshold))/S

print(expectation2)

hist((2\*pi\*temp)^(sqrt(2)),breaks=10,xlab="Bonus",main="Histogram of Bonus in Week 2")

1. http://en.wikipedia.org/wiki/Gamma\_distribution#Summation [↑](#footnote-ref-1)
2. http://www.proofwiki.org/wiki/Sum\_of\_independent\_Poisson\_random\_variables\_is\_Poisson [↑](#footnote-ref-2)